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ABSTRACT

Part of the Fastback series from Phi Delta Kappa, this publication provides background information and several educational applications of modeling for use with students from elementary school to college. It contains a rationale for employing mathematical modeling in instruction, specific examples, strategies for grouping students using technology and structuring learning, and a discussion of student assessment in mathematical modeling. Mathematical modeling helps student develop mathematics literacy by enabling them to exchange ideas, learn from one another, identify problems, define problems, and create hypothetical problems. Contains 53 references. (DDR)

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FASTBACK®

Mathematical Modeling

Eileen Veronica Hilke

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Mathematical Modeling

by
Eileen Veronica Hilke

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Why Use Mathematical Modeling

Imagine a math class where small groups of students work on a problem involving a trip from Green Bay, Wisconsin, to Chicago. They need to determine the amount of gas that a minivan will use on a trip to a football game in Chicago and back. The problem requires them to use map skills to locate the best route to the stadium. Students will have to calculate the number of miles they will travel, find out the minivan's gas consumption, and check the price of gas in order to solve the problem.

This real-world problem encourages students to actively discuss mathematics. The math they use will flow from the problem; and the problem may be approached in various ways, depending on the thinking processes and the mathematics backgrounds of the students in the group. This approach to mathematics is called mathematical modeling.

According to Peterson, Fennema, and Carpenter, "Learning is the making of connections between new information and the learner's existing network of knowledge — the construction of knowledge by the learner — and instruction should facilitate these connections" (1988-89, p. 43). To encourage the formation of cognitive networks, students must be taught new information in a meaningful way. Taking what the learner already knows and linking it to new information is crucial, and modeling is an excellent way to accomplish this linkage. (For a general overview of constructivist teaching, see fastback 390 *Constructivist Teaching* by John A. Zahorik.)

Mathematical modeling is part of a growing reform movement in mathematics instruction. Koss and Marks believe that this reform effort:

> grounded in a constructivist view of learning, fosters growth in each student's mathematical thinking, through active exploration, communication of ideas, and reflection over an extended period. To achieve this goal, a teacher may acknowledge various learning styles, present worthwhile tasks and time to explore mathematical ideas, emphasize making sense of the mathematics, encourage each student to express his or her own thinking and share it with peers, and use the teacher's knowledge of students to design instruction that will lead them toward greater understanding. (1994, p. 616)

Students in the same grade and class possess various prior knowledge and use various learning styles. Thus, contend Rowan and Bourne (1994), "each child will interpret and connect ideas differently and must construct for himself or herself the connections and relationships among those concepts essential to understanding mathematics" (p. 25).

Dossey contends: "Mathematical modeling is the process by which real-world situations are represented in mathematical terms. Many problems can be solved by creating a mathematical model, manipulating the model, interpreting the possible solutions, and validating them in the original problem situation" (1990, pp. 3-4). Therefore, in mathematical modeling, the students themselves determine how to structure the problem — what model to create.

Unlike traditional math instruction that can become boringly formula-dependent and disconnected from real-world experience, mathematical modeling focuses on solving problems that have true-to-life applications. According to Wirtz and Kahn: "Problem solving, in its mathematical sense, is a limited term that refers to activities that involve uncertainty, and require the use of reflective thought, trial and error, evaluation, decision-making, and other high-level cognitive skills, attitudes and behaviors. Application is a broader label that may include all uses of mathematics, from simple computations to grocery-store arithmetic, as well as full-blown problem solving" (1982, p. 21).

Several years ago, when the National Council of Teachers of Mathematics (NCTM) developed new curriculum and evaluation standards for school mathematics, they set five general goals for all students: 1) to value mathematics, 2) to become confident in their ability to do mathematics, 3) to become mathematical problem solvers, 4) to learn to communicate mathematically, and 5) to learn to reason mathematically (NCTM 1989, p. 5). These goals are clearly exemplified in mathematical modeling.

Mathematical Modeling in Action

For students at all levels, teachers can begin instruction in mathematical modeling by teaching five simple, commonsense steps for solving most problems. When students understand these steps, they will find it easier to understand how to approach a problem using modeling. These five steps include:

1. Identify relevant information for solving the problem.
2. Choose the mathematical concepts that will be needed to solve the problem, then outline the procedure for determining the solution.
3. Solve the problem using appropriate mathematical calculations.
4. Check the accuracy of the work and decide on the implications of the mathematical solution.
5. Review and discuss the process.

Real-world problems often are not clearly presented. Thus, to begin, the student must learn to identify the information that is relevant and discard extraneous information. Having winnowed to the essential problem, the student's next step is to choose the math concepts and operations that will lead to solving the problem and put them into a logical order. The next steps are to follow that order and solve the problem, and to then check the accuracy of the work. Finally, the student can review his or her procedures and discuss them with other students, who may have used other procedures for arriving at a solution. Indeed, there

may be other solutions, because many real-life problems can be interpreted in more than one way.

Talking through the process assists students in articulating their thoughts about how to solve a problem. Communication skills are enhanced. Students may have various ideas about how to define a problem before tackling its solution. Therefore, the instruction also may develop skills in making decisions and working cooperatively with others. Each developmental level will use mathematical modeling in its own way.

Elementary School

In the elementary school years, teachers should discuss the thinking process for defining and solving a problem before, during, and after working through the problem. After a while, the students will feel comfortable talking through their own thought processes when analyzing a problem.

Teachers should select problems that match their students' developmental level and use concrete operations, manipulatives, and visuals. For young children in particular, manipulative materials help to bring meaning to the concepts they are trying to learn by making abstract ideas concrete. Manipulatives are helpful for most students, but particularly for children who learn best through kinesthetic, or hands-on, activities. Effective manipulatives can be simple objects, such as beans, popcorn, macaroni, and paper clips. Some manipulatives are available commercially.

Readers will note that these procedures echo the NCTM standards for students in grades K-4. Rowan and Bourne advise teachers to:

1. Provide meaningful situations in which children can solve problems through the use of manipulative materials, interesting activities, and real-life situations.
2. Provide ample time for children to construct their own knowledge, reflect on their activities, and exchange ideas with peers and interested adults.

3. Encourage children to explain their thinking and how they arrived at a solution. Support their methods and ideas through thoughtful questioning. (1994, p. 132)

Rowan and Bourne continue: "Make it a priority to have students rely on themselves to solve mathematics problems, to be mathematicians. The goal is, after all, math power for all students" (p. 132).

Following is an example of mathematical modeling at the elementary level.

> Problem: One hundred students participate in a soccer tournament. To demonstrate good sportsmanship, they all want to shake hands at the end of the tournament. How many handshakes will there be?

In defining this problem, the students discuss the basic understandings: 1) each of the 100 students will shake hands with 99 other students, and 2) the solution to this problem will be the total number of handshakes.

Various procedures may be used to arrive at the solution. For example, a group of advanced students might arrive at a formula:

$$\frac{N(N-1)}{2}$$

By applying this formula, they would solve the problem:

$$100 \times (100-1) \div 2 = 4,950.$$

Another group of students might make a chart:

Soccer Players	Handshakes
1	0
2	1
3	3
etc.	etc.

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And yet another group of students might act out a solution by having 10 students shake hands and then discussing how that reasoning would apply to 100 students.

Students also can be encouraged to create their own problems when teachers integrate children's literature into the math curriculum. Students read a variety of trade books, and so the teacher can develop a center for books that focus on math. Students can use those resources in developing problems, for which their classmates find the solutions. Some appropriate selections include:

Hoban, Tana. *26 Letters and 99 Cents*. New York: Scholastic, 1987.

Students can view the vivid pictures, then create math problems that combine the coins pictured on several pages. Real money also can be used as manipulatives. Students can design their own mini-books by doing artistic coin rubbings and writing math problems. For a social studies focus, students can find out about coins used around the world and determine their worth in comparison to U.S. coins.

Hutchins, Pat. *The Doorbell Rang*. New York: Mulberry Books, 1986.

The theme of sharing cookies with friends also shows how division is used. Children start with six cookies each, then they keep dividing them as more friends visit. For a project, students can take objects and determine division problems by setting up a situation similar to that in the book. They also could look at a cookie recipe and think about how to increase ingredients to have enough food for 100 people, or decrease the recipe for two people.

McGrath, Barbara Barbieri. *The M&M's Brand Counting Book*. Watertown, Mass.: Charlesbridge, 1994.

Through colorful pictures of M&M candy, students learn to count and combine numbers. Students might be given a pack of candy as manipulatives. They can think of combination problems and graph the colors found in each pack.

Schwartz, David. *How Much is a Million?* New York: Scholastic, 1985.

Have students think of ways to explain how much a million is in their own words. Develop a bulletin board using the student papers. Encourage them to write a mini-play and act out their way of explaining a million to another character. Then videotape the creations.

Clearly related to mathematical modeling at the elementary level is Cognitively Guided Instruction (CGI), a popular teacher training model used for assessing how children develop math thinking processes. The CGI program was researched by Elizabeth Fennema and Thomas P. Carpenter at the University of Wisconsin-Madison. This approach enables students to work with a variety of problems and use different strategies to solve these problems. Jenkins and Keith comment: "Teachers who use the principles of CGI encourage students to build upon their natural math strategies. Story problems lead to a deeper understanding of math. Students are also encouraged to listen to each other, ask questions and explain how they solve their problems. The emphasis of CGI is on the process vs. the product" (1991, p. 5).

This emphasis strongly contrasts with more traditional mathematics instruction, where the emphasis is on the correct answer. Jenkins and Keith continue: "The way to accomplish the goals of our mathematics curriculum is for children to understand what is taught. Recall of facts will become automatic if children are allowed to discover and apply the interrelationships of numbers. Children will be at different stages of development and will progress at different rates" (p. 9). The general conceptual approaches of mathematical modeling, or the specific concepts of CGI instruction, show students that mathematics is useful and challenging. These approaches also build students' confidence in their math abilities.

Middle School

Teachers in grades 5 through 8 usually are working at moving their students from concrete thinking to abstract thinking in mathematics.

Introducing algebra can be an important part of this transition. The use of algebra is a good way to present real-world situations. Although many algebra concepts will not be mastered until high school, middle school is a good time to present basic ideas and to encourage higher-level thinking.

Middle-level students include a wide range of developmental abilities; therefore, they can share insights and learn from each other. Teachers should encourage students to use the talk-through method when explaining mathematical processes to each other. Vetter believes:

> Peer teaching talk-throughs enable students to build a learning network, including the following benefits: 1) a reinforcement of correct mathematical thought processes, 2) a more effective understanding of just how mathematics works for us, 3) a lessening of the feelings of frustration and defeat sometimes associated with mathematics, 4) an opportunity to extend mathematics concepts through discussion and application, and 5) the empowerment of an investigative partnership. (1992, p. 168)

Professional journals are a good source for locating supplementary items to complement a mathematics program. The following examples, compiled by William Jamski (1990), appeared in *Arithmetic Teacher*. (In 1994, *Arithmetic Teacher* was renamed *Teaching Children Mathematics and Mathematics Teaching in the Middle School*.)

> Old McDonald
>
> A farmer has hens and rabbits. Together these animals have eight heads and 22 feet. How many of each did the farmer have? (Answer: three rabbits and five hens.) (p. 13)

> Koala Klimb
>
> A sleepy koala wants to climb to the top of a eucalyptus tree that is 10 meters tall. Each day the bear climbs up 5 meters, but at night, while asleep, it slides back 4 meters. At this rate, how many days will it take the bear to reach the top of the tree? (Answer: 6 days.) (p. 13)

An effective program that uses mathematical modeling is the Middle Grades Mathematics Project (MGMP), a curriculum program for

grades 5 through 8. MGMP was developed at Michigan State University with funding from the National Science Foundation. Lappan, Fitzgerald, Winter, and Phillips characterize the program this way: "The goal of the MGMP materials is to help students develop a deep, lasting understanding of the mathematical concepts and strategies studied. MGMP materials concentrate on a cluster of important ideas and the relationships that exist among these ideas" (1986, p. 1).

The MGMP activities are built around a mathematical challenge. Instruction has three phases:

1. The teacher launches a challenge of new concepts and definitions.
2. Exploration is done individually or in small groups and includes data gathering, idea sharing, and problem-solving strategies.
3. The final phase is summarizing.

A similar but newer program from Michigan State University, the Connected Mathematics Project, provides connections between mathematics and other disciplines. Teaching activities also connect with the interests of students and the world outside school. Thematic units help teachers draw lessons from the math standards (Jacobson 1995).

Another successful middle school project, Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR), uses a hands-on, concrete, visual approach (Graves 1995).

High School and College

High school and college students are more sophisticated problem solvers; but as much as younger students, they need mathematical modeling that connects abstract concepts to concrete situations. High school teachers still may wish to pose problems and ask individual students or cooperative groups to analyze and ultimately solve the problems. But many students at both high school and college levels can choose their own real-world problems to analyze. The problems can be drawn from a variety of academic disciplines.

Swetz and Hartzler comment that "mathematical modeling is a systematic process that draws on many skills and employs the higher cognitive activities of interpretation, analysis and synthesis" (1991, p.2). As students work on their real-world problems, they should debate and challenge one another's ideas. Because many problems can be solved in numerous ways, seeking solutions should be approached with an open mind. Students need to look at different ways of approaching each task. They also should be encouraged to select problems that hold a special interest for them and to develop a variety of solution models.

Following is a sample high school problem that asks students to take a typical school situation and find ways to apply their knowledge of mathematics. The problem also draws on skills in other areas, such as research and communication.

> Problem: Some students are complaining that a 30-minute lunch period is not long enough because sometimes they have to stand in a salad bar or hot lunch line almost 20 minutes, eat in 10 minutes, and then run through the halls so they won't be late for the start of the next class period. The administration has agreed to investigate the problem.

This problem presents opportunities for research, collaboration, and mathematical application. For example, some students may want to study the circumstances that result in long lines. Are there long lines every day, or just on days when pizza or other favorite meals are on the menu? If the problem is persistent regardless of the menu, where does the problem start — too many students, too few staff, or insufficient space? Ratios of staff to students, students to servings stations, and so on, can be determined. Different groups may define the problem in different ways, which will lead to collaboration and debate.

Mathematical modeling presents a wealth of possible situations that relate to students' personal and school lives. Following are two more sample problems:

Problem: Tom would like to have a CD player, but he isn't sure if it would be more economical to rent or buy one. Because he does not have cash for an outright purchase, he would need to pay for a player by borrowing money, which would require interest. His other option is to rent a CD player. Which is the better deal and under what circumstances?

Problem: Students open a school store to sell T-shirts with logos of their athletic teams. They want to set a good price for the T-shirts so that they can make enough money to sponsor a homecoming dance. Some of the students believe that they will sell more shirts if they keep the price low; others believe that the market is limited and so the price should be kept as high as possible without driving away customers. What is the "optimum" selling price and what is the rationale behind that price?

At all academic levels, students should use the talk-through method. Not only is this method helpful to students' understandings, but it also can help the teacher to diagnose how well students are able to analyze a problem.

A popular program for both middle school and high school is the University of Chicago School Mathematics Project (UCSMP). This is a program for grades 7 through 12 in which a four-year high school math curriculum is taught over six years. According to Usiskin and colleagues, UCSMP "prepares students to use mathematics effectively in today's world, promotes independent thinking and learning, helps students improve their performance, [and] provides the practical support you need" (1992, p. T4).

The project also encourages the use of such technology as calculators and computers, incorporates real-world data, and provides students with the ability to deal with real-world problems.

A new course, *Pacesetter Mathematics: Precalculus Through Modeling* for fourth-year mathematics students, has been developed and is being pilot tested by 500 students in 10 states in the hopes of raising education standards. The course follows the National Council

of Teachers of Mathematics standards and is co-sponsored by Educational Testing Service and the College Board. According to Gams:

> The course is centered on mathematical modeling because modeling requires students to be actively involved in shaping the problem and developing solutions. The course uses real-world situations so that students can understand how mathematics can help them make sense of the world around them. In a typical modeling situation, students are presented with a problem, asked to look at alternatives, develop mathematical explanations, describe the situation, and predict what might happen in the future. (1994, pp. 2-3)

Some examples of modeling problems in this course include:

- Estimating when the world's population will reach 10 billion.
- Determining hours of daylight anywhere in the world.
- Measuring bones to determine height.
- Charting the course of a hurricane.

Teachers assess student learning through projects and performance-based tasks. Teachers also can check progress by reviewing journals, essays, and portfolios to see how the students are recording their own progress. The course also includes many opportunities for oral communication.

The Role of the Teacher in Mathematical Modeling

The teacher in mathematical modeling must be a facilitator and guide. Usually, the teacher begins the modeling process by posing a problem or by helping students to develop and state a problem. Next, the teacher asks individual students or cooperative groups to analyze and then solve the problem. Then the teacher asks, "Did anyone have a different way of structuring and solving this problem?" This last step is key. The modeling process, in most instances, will result in different students or groups developing different models for solutions.

Often, different solutions can be found for the same problem. The teacher should welcome these differences and encourage lively discussion that leads students to analyze their definitions of problems and their thoughts and strategies for solutions. By approaching mathematical modeling in this manner, the teacher departs from traditional direct instruction that is focused on finding the one correct answer, but rather generates higher-level thinking that values different correct answers according to various problem interpretations.

When the teacher guides and facilitates learning, students become self-reliant in completing math projects. The teacher provides necessary information as students work up to their ability in a more independent manner. Of course, the teacher understands that some students may not have the necessary prior skills to complete a project; therefore, some basic, direct instruction may be used to provide the foundation for independent problem solving.

The teacher also makes connections between mathematics and other subjects. According to Jacobs, three practical ways to start integrating mathematics include: 1) focusing initial efforts on one carefully conceived multidisciplinary unit, 2) developing units around subjects that naturally overlap, and 3) conducting school-based research to determine what is actually taught (1993, p. 301). A particularly suitable integration is mathematics and science. "The methodology of mathematical inquiry shares with the scientific method a focus on exploration, investigation, conjecture, evidence, and reasoning" (Mathematical Sciences Education Board 1990, pp. 44-45).

An example of one program that combines mathematics and science is AIMS, the acronym for Activities Integrating Mathematics and Science. AIMS is a holistic approach that encourages higher-level thinking, such as hypothesizing and generalizing. The goal is "to provide hands-on learning through real world experiences, foster creative and divergent thinking and discussion, and provide opportunities for application" (Berlin and Hillen 1994, p. 284).

Another example of a curriculum connection is to integrate language arts and math by encouraging students to write about math topics, such as procedures and reasoning that they have used to solve problems. Such writing documents growth in self-learning. According to Dusterhoff (1995), integrating writing and mathematics has several advantages:

> 1. Math topics and experiences provide interesting and challenging material about which to write, 2. Writing in mathematics increases mathematics learning, 3. General writing ability improves through writing in mathematics, 4. Writing can spark interest in the study of mathematics, 5. Writing helps students explore, clarify, confirm and extend their thinking and understanding, 6. Writing helps the teacher assess student learning and plan future instruction. (1995, pp. 48-49)

Keeping a math journal is a particularly apt learning tool. In a narrative format, students reflect on their math progress. Teachers can stimulate journal entries by posing questions, such as, What did you

learn about your math skills? What problems surfaced when you were determining a strategy to use? Teachers can ask students to explain in writing the steps they used to arrive at a particular solution. Affective questions also can be asked: How did you feel about this problem? Does the problem and its solution reflect your experience? Journal writing "enables students to use the skills of reading, listening, viewing, and questioning to interpret and evaluate mathematical ideas" (Norwood and Carter 1994, p. 148). The writing experiences will help make students more mathematically literate and will help them recognize how important math is in their daily lives.

Grouping Students

When students work on modeling problems, they need to work together in order to discuss their thinking and solution strategies. Such discussion encourages students to consider a variety of viewpoints. By analyzing these viewpoints and deciding how to approach a problem — which mathematical procedures to use, the type of solution to seek — students use higher-level thinking and learn to work collaboratively.

In fact, cooperative learning is an excellent approach to group work in mathematical modeling. In cooperative learning, three to five students work together in small groups that draw on individual strengths. The teacher sets up the groups, and the groups stay together until the task is complete. Cooperative learning fosters academic cooperation among students, encourages positive group relationships, develops students' self-esteem, and enhances academic achievement. (For more information, see fastback 299 *Cooperative Learning.*)

In all cases, students need to be taught how to work in groups. The teacher also should explain his or her expectations for the students as individuals and for the group as a whole. Smith (1993) offers these guidelines for typical problem-solving groups:

1. Groups formulate and solve problems. Each group places their formulation and solution on an overhead transparency

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or on paper, and ensures that each member understands and can explain it.
2. Randomly selected students invited to present their group's model and solution.
3. Discussion of formulation and solution. All members of the class are expected to discuss and question all models. The discussion alternates between whole class and small group.
4. Groups process their effectiveness in working together as a team.
5. Each group prepares and submits a homework assignment report.

Using Technology

Students can use calculators to discover number relationships and to explore solutions to problems. In fact, such use is advisable, according to the Mathematical Sciences Education Board, which says,

> Research suggests that access to calculators in a well planned program of instruction is not likely to obstruct achievement of skill in traditional arithmetic procedures. More optimistically, it appears that when students have access to calculators for learning and achievement testing, they perform at significantly higher levels on both computation and problem solving. In particular, students using calculators seem better able to focus on correct analysis of problem situations. (1990, p. 23)

Calculators should be used starting at the primary level. But teachers at all levels must make sure that students understand when calculations should be done mentally or with pencil and paper and when a problem should be solved using a calculator. For example, middle school students might be asked to use mental math to add up the school supplies that can be purchased for $10.00. But to determine the amount of carpeting needed for a classroom, students may be encouraged to calculate square footage and cost per square foot using a calculator.

For upper-level students, graphing calculators are becoming a valuable tool for mathematical modeling. Graphing calculators are hand-held

calculators with a small screen that can show a graphic representation of an equation. These calculators eliminate time-consuming pencil-and-paper graphing and thus allow students to have more experiences with a wide variety of graphs and graphing problems.

Computers, too, are becoming a staple in math instruction. According to Demana and Waits:

> Computer generated numerical, graphical, and symbolic mathematics is revolutionizing the teaching and learning of mathematics. The computer can be a desktop computer with a computer algebra system or a pocket computer with software built-in (a graphing calculator). The content of mathematics is changing. Reduced time is spent on paper and pencil methods and increased time is spent on application, problem solving, and concept development. Instructional methods are also rapidly changing. Investigative, exploratory methods are becoming more common in mathematics courses. (1993, p. 1)

Computers are important, and students should know how to use them effectively. They can be particularly useful when teachers make a conscious effort to move students beyond drill and practice programs to more complex problem situations. Computers can assist students to develop higher-level thinking — analysis, synthesis, and evaluation.

Building Strategies

As a guide and facilitator in mathematical modeling, the teacher's most important role is structuring learning so that all students acquire a repertoire of learning strategies. A strategic learner is one who assesses the learning situation, defines what is to be learned, and chooses ways to learn that meet his or her needs.

Much of the practical work with learning strategies has been done in the context of reading instruction and can be imported into mathematical modeling contexts to increase students' abilities to understand real-world problems and how to solve them. Three such learning strategies are:

1. Directed Reading Thinking Activity (DRTA). The teacher guides the students to read a problem, ask questions about the problem, make predictions about a solution or set of possible solutions, and then validate or reject those predictions by solving the problem.
2. KWL. The teacher helps the students make a chart for analyzing their problem and how they solve it. The chart has three columns, in which students write what they Know, what they Want to know, and what they Learn.
3. Mapping. The teacher assists students to define the central problem or concept and then visually connect related ideas or processes. For example, the teacher might put the word "quadrilateral" in the center of the chalkboard to serve as the hub of the map and then ask students to give examples of types of quadrilaterals. The examples — "square," "rectangle," "parallelogram," etc. — would be written in bubbles connected by spokes to the hub. Later, students might connect definitions to the examples, such as "four equal sides" for "square."

Many such strategies can be directly taught initially. Later, after the students have built a repertoire of available learning strategies, they will be able to choose the best strategy to use in various situations.

Assessment in Mathematical Modeling

There are various ways to assess students in mathematics. Conventional standardized tests provide one view of whether students have mastered basic skills. This type of information can be important to the general public, school administrators, and school boards. However, standardized assessments do not match the specific curriculum of a school district. Thus it also is important for school personnel, students, and parents to see academic accomplishments according to the specific objectives of their own school district. Alternative forms of assessment are important for this purpose.

As the methods of teaching math change, the assessment procedures also must change. Today, the math curriculum in most school districts emphasizes solving real-world problems, not just making calculations or using formulas. As a result, student accomplishments need to be evaluated in more realistic and meaningful ways. So-called authentic assessment is one of these ways.

Authentic assessment better assesses how students define or formulate problems, develop hypotheses, and reach solutions. Walberg, Haertel, and Gerlach-Downie believe, "Assessments of students' subject matter expertise should consider: 1) the level of detail used to represent a problem, 2) the characteristics of the problem, 3) the conceptual skills and principles used, 4) the degree of organization and flexibility in reasoning, and 5) the selection and execution of solution strategies" (1994, p.15). Several forms of authentic assessment incorporate these points.

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One form of authentic assessment is the use of open-ended questions. In 1987-88 these types of questions were included in the grade-12 California Assessment Program (CAP). The open-ended questions were intended to provide students an opportunity to think for themselves, to have students construct their own responses and demonstrate understanding of a problem, to encourage many ways to solve problems, and to have students apply appropriate mathematical tools to situations (California Assessment Program Staff 1989, pp. 1-2).

Following is an example of an open-ended question:

> James knows that half of the students from his school are accepted at the public university nearby. Also, half are accepted at the local private college. James thinks that this adds up to 100%, so he will surely be accepted at one or the other institution. Explain why James may be wrong. If possible, use a diagram in your explanation. (California Assessment Program Staff 1989, p. 58)

Two ways of scoring open-ended questions are termed *analytic* and *holistic*. To score a student's response to an open-ended question analytically, the evaluator considers the response in relation to specified criteria. These criteria are usually constructed as a checklist or scoring guide, termed a *rubric*. Points are awarded according to how well the student's response meets each criterion. Usually, the individual criterion scores are added together for a composite score. In holistic scoring, the evaluator's judgment is based on the student's response as a whole, rather than on specific features of the response. A more generalized rubric is developed for holistic scoring, and each student's response is given a single score.

Portfolio assessment is another method of authentic assessment, in which evaluation is based on actual (authentic) samples of the student's work. A student's portfolio is a folder that contains a variety of student-selected work, such as projects, journal writing, modeling examples, selections from daily assignments, and other items. Although the material in the folder is student-selected, the teacher can establish criteria for materials to be included in the portfolio. For example, the teacher

might require the student to include a biography of a historical figure in mathematics, certain research papers, or a project that uses technology. Students also may be required to reflect on the scope of their portfolio and to record their reflections in writing or to provide a rationale for choosing the various sample works to include in the portfolio.

Portfolios are an excellent tool for self-assessment. Asturias comments: "The self-assessment aspect makes students aware of areas that need improvement. Students gain a deeper understanding of the concepts they are learning and are able to communicate better mathematically" (1994, p. 701). Thus, by employing portfolio assessment, teachers also can help students feel pride about their accomplishments, which will enhance their self-esteem and encourage a positive attitude toward mathematics.

As in the case of analytically scored, open-ended questions, a scoring rubric (or set of specific criteria) is important for evaluating the math portfolio. Rubrics should be tailored to the course curriculum for which they are used. Following are 10 general criteria that can be used or adapted as a checklist for evaluating elementary math portfolios. Each criterion might be rated "uses often," "starting to use," "not using," or "not applicable." Or the teacher might simply rate each criteria using a number from 1 (low) to 5 (high).

Elementary Math Portfolio Checklist

1. Uses a variety of problem-solving steps.
2. Chooses appropriate solution strategies.
3. Constructs suitable equations.
4. Can solve problems independently.
5. Contributes to solving problems in a group.
6. Selects relevant information.
7. Checks solutions for accuracy.
8. Completes math journal entries.
9. Completes self-assessment.
10. Keeps math portfolio up to date.

Conclusion

Mathematical modeling has been around for centuries. It is how people solve problems in everyday life. The concept is newer as an instructional strategy.

Both common sense and recent research assert that students learn more effectively when what and how they learn is connected to their lives. Most mathematical concepts, while they can be viewed as abstractions, have concrete, practical applications. Math works for us when we add up a list of charges on a bill, make change at the grocery, or calculate how much carpet we need for a bedroom. Math also works for children when they want to divide jelly beans or figure out how much rope they will need to make four jump ropes of a certain length.

The NCTM Standards state that "today's society expects schools to ensure that all students have an opportunity to become mathematically literate, are capable of extending their learning, have an equal opportunity to learn, and become informed citizens capable of understanding issues in a technological society. As society changes, so must its schools" (1989, p. 5). Mathematical modeling can help teachers meet these goals.

Through mathematical modeling, students "talk math." They exchange ideas, learn from one another, and become mathematically literate. They learn to identify problems, to define problems, even to create hypothetical problems — and then they learn how to solve them, often in several different ways. And, because their school mathematics

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problems mirror real life, students become better able to carry the knowledge they gain through mathematical modeling into their lives outside the school and thus become adept at solving the problems humans face every day.

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